Supplementary exercise 6.140 of IPS7e

Dimethyl sulfide (DMS) thresholds for 10 beginning students of oenology. We set up the notation and assumptions as follows:

Data: $X_1, \ldots, X_{10}$.

Model: $X_1, \ldots, X_{10}$ are i.i.d. (a simple random sample, SRS) and normally distributed $\mathrm{N}(\mu, \sigma)$, where μ is unknown, and $\sigma = 7\ \mu g/l$ is assumed known.

(a) Some descriptive statistics from Minitab (including the stemplot, which is a much better representation than a histogram of the distribution for such a small sample):

Descriptive Statistics: odor

Statistics

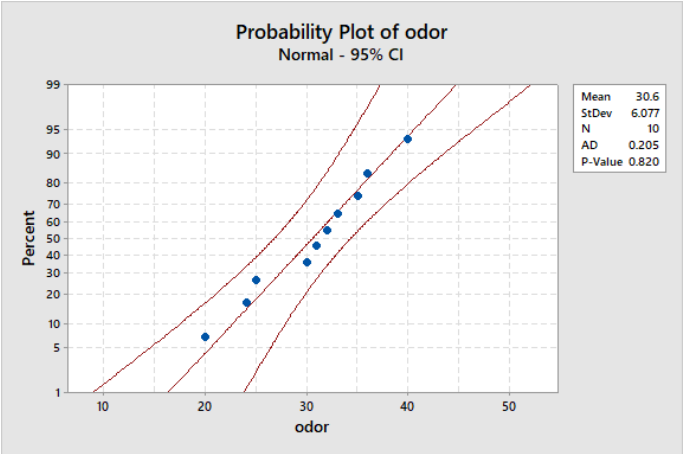
Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
odor	10	0	30.60	1.92	6.08	20.00	24.75	31.50	35.25	40.00

Stem-and-Leaf Display: odor

Stem-and-leaf of odor N = 10

2	2	04
3	2	5
(4)	3	0123
3	3	56
1	4	0

Leaf Unit = 1



Comments:
The distribution looks indeed roughly symmetric with no outliers, although it is difficult to assess distribution shape from such a small sample. The points in the normal probability plot are close to the straight line, and the P-value of the Anderson-Darling normality test is 0.82. There is no evidence against a normal distribution of the data.

(b) We will use the Minitab menu (`Basic Statistics-1 Sample Z`) to compute the 95% confidence interval; it equals $(26.3, 34.9)$.

One-Sample Z: odor

Descriptive Statistics

N	Mean	StDev	SE Mean	95% CI for μ
10	30.60	6.08	2.21	(26.26, 34.94)

μ: mean of odor
Known standard deviation = 7

(c) We set up the hypotheses as follows,

$$\mathrm{H}_0 : \mu = 25 \ \mu g/l \quad \text{versus} \quad \mathrm{H}_a : \mu > 25 \ \mu g/l.$$

We chose a one-sided alternative hypothesis because interest is only in a higher threshold for the untrained students. Also here, a two-sided alternative hypothesis could be justified as well. We go back to the same Minitab menu for the calculation of a z-test; note that the alternative hypothesis needs to be set under `Options`.

One-Sample Z: odor

Descriptive Statistics

N	Mean	StDev	SE Mean	95% Lower Bound for μ
10	30.60	6.08	2.21	26.96

μ: mean of odor
Known standard deviation = 7

Test

Null hypothesis	H_0: μ = 25
Alternative hypothesis	H_1: μ > 25

Z-Value	P-Value
2.53	0.006

Comments:
The P-value equals 0.006, so we have fairly strong evidence to reject H_0, and therefore conclude that the data indicate that students (in general, not just these 10 students) indeed have a higher DMS odor threshold.

Finally a more (Minitab) technical note: when we use a one-sided alternative for the test, the listing also displays a one-sided confidence interval (CI). In the course, one-sided CIs are not discussed, and it is therefore recommended to simply ignore the information about CIs when a one-sided alternative is used. If both a CI and a test with a one-sided alternative hypothesis are desired, one will therefore have to use the menu twice.