## Supplementary exercise 4.71 of IPS7e

A discrete probability distribution for the temperature ($X$) of the flame in a glass heating process. (As an aside, one might think that a continuous distribution would be more natural.) The probability function is given by the values in the table below. We note that the probability distribution is valid because the probabilities are $\geq 0$ and sum to 1 across the possible outcomes.

| Temperature $x$ (°C) | 540 | 545 | 550 | 555 | 560 |
|---|---|---|---|---|---|
| Probability $p(x)$ | 0.10 | 0.25 | 0.30 | 0.25 | 0.10 |

(a) We calculate the mean temperature as (letting $X$ denote a random variable from this distribution):

$$\mathrm{E}\,X = \sum_x x \cdot p(x) = 540 \cdot 0.1 + 545 \cdot 0.25 + 550 \cdot 0.3 + 555 \cdot 0.25 + 560 \cdot 0.1 = 550.$$

The distribution is (visibly) symmetrical around 550, so we could have guessed that the mean would equal 550.

We calculate the standard deviation for $X$ in two steps: first the variance, and then the standard deviation:

$$\begin{aligned}
\mathrm{Var}\,X &= \sum_x (x - \mathrm{E}X)^2\, p(x) \\
&= (540-550)^2 \cdot 0.1 + (545-550)^2 \cdot 0.25 + (550-550)^2 \cdot 0.3 \\
&\quad + (555-550)^2 \cdot 0.25 + (560-550)^2 \cdot 0.1 \\
&= 10 + 6.25 + 0 + 6.25 + 10 = 32.5, \\
\mathrm{sd}\,X &= \sqrt{\mathrm{Var}X} = \sqrt{32.5} = 5.7.
\end{aligned}$$

(b) The new variable of interest is $T = X - 550$, i.e., a translation of the original variable X by the value (–550). We can either use the rules for a translation, or the general rules for linear transformation of a random variable.

$$\begin{aligned}
\mathrm{E}\,T &= \mathrm{E}X - 550 = 0 \quad \text{(i.e., we adjust the center by the translation),} \\
\mathrm{sd}\,T &= \mathrm{sd}X = 5.7 \quad \text{(i.e., no impact on spread by a translation).}
\end{aligned}$$

If using the formulae for means and standard deviations/variances of random variables in Lecture 3, you should set $b = 1$ and $a = (-550)$.

(c) Here the variable of interest is $Y = 1.8 \cdot X + 32$. We should use the mean and standard deviation rules for linear transformation of random variables, with $a = 32$ and $b = 1.8$.

$$\begin{aligned}
\mathrm{E}\,Y &= 32 + 1.8 \cdot \mathrm{E}X = 32 + 1.8 \cdot 550 = 1022, \\
\mathrm{sd}\,Y &= 1.8 \cdot \mathrm{sd}X = 1.8 \cdot 5.7 = 10.26 \approx 10.3.
\end{aligned}$$

We finally, on the next page, show Minitab commands to carry out these calculations after the distributions has been entered in suitable columns, labeled `x` and `p(x)`. If you use the provided data file, the first commands for the data entry are no longer needed, but you may want to rename the columns to simplify the notation.

Minitab commands and output:

```
Name c1 "x"
Set 'x'
 1( 540 : 560 / 5 )1
 End.
name c2 "p(x)"
Set 'p(x)'
 1( 0.1 0.25 0.3 0.25 0.1 )1
 End.
Name C3 'meanx'
Let 'meanx' = 'x' * 'p(x)'
Sum 'meanx'.
Name C4 'varx'
Let 'varx' = ('x'-550)^2 * 'p(x)'
Sum 'varx'.
Name C5 'xF'
Let 'xF' = 1.8 * 'x' + 32
Name C6 'meanxF'
Let 'meanxF' = 'xF' * 'p(x)'
Sum 'meanxF'.
Name C7 'varxF'
Let 'varxF' = ('xF' - 1022)^2 * 'p(x)'
Sum 'varxF'.
Print 'x' 'p(x)' 'meanx' 'varx' 'xF' 'meanxF' 'varxF'.
```

**Sum of meanx**

Sum of meanx = 550

**Sum of varx**

Sum of varx = 32.5

**Sum of meanxF**

Sum of meanxF = 1022

**Sum of varxF**

Sum of varxF = 105.3

## Data Display

### Data

| Row | x | p(x) | meanx | varx | xF | meanxF | varxF |
|---|---|---|---|---|---|---|---|
| 1 | 540 | 0.10 | 54.00 | 10.00 | 1004 | 100.40 | 32.40 |
| 2 | 545 | 0.25 | 136.25 | 6.25 | 1013 | 253.25 | 20.25 |
| 3 | 550 | 0.30 | 165.00 | 0.00 | 1022 | 306.60 | 0.00 |
| 4 | 555 | 0.25 | 138.75 | 6.25 | 1031 | 257.75 | 20.25 |
| 5 | 560 | 0.10 | 56.00 | 10.00 | 1040 | 104.00 | 32.40 |

Note that the Minitab calculator cannot be used for a simple (scalar) calculation such as: $\sqrt{105.3} = 10.25$. You would either have to use the calculator built into Windows or another calculator.