

Mid-term exam, 24 October 2024

All aids are allowed, except a computer and personal assistance, as well as the use of any external information pertaining to the specific data and questions. Restricted use of some computer-like devices (including laptops, tablets and smartphones) is permitted under the rules described at the VHM 801 course homepage. The exam consists of one question with five parts, or subquestions (labeled by letters **(a)** – **(e)**), which should all be answered.

The mid-term exam accounts for 15% of the course mark; however, every student may choose to waive the result of the mid-term exam. The duration of the mid-term exam is 1 hour.

Generally, **statistical models and methods should be specified**, and all statistical analyses should be summarized in conclusions.

Question 1. (15 points)

Skin from cadavers can be used to provide temporary skin grafts for severely burned patients. The longer such a graft survives before its inevitable rejection by the immune system, the more the patient benefits. A medical team investigated the usefulness of matching grafts to patients with respect to the HL-A antigen system. Each patient received two grafts, one with close HL-A compatibility, and the other with poor compatibility. The rejection times of the skin grafts (that is, the time until each graft was rejected) are shown in the table below.

Rejection time (days)	Patient										
Compatibility	1	2	3	4	5	6	7	8	9	10	11
close	37	19	57+	93	16	23	20	43	29	60+	18
poor	29	13	15	26	11	18	26	43	18	42	19

In the table, a “+” indicates an incomplete observation (e.g., 57+ means that the rejection time was at least 57 days). Patient 3 died with a graft still surviving, and the observation on patient 10 was incomplete for an unspecified reason.

Use the information about the study, the data in the table and the descriptive statistics listed at the end to answer the questions below. Note that the construction of several new variables from the variables `close` and `poor` containing the rejection times for grafts with close (poor) compatibility is described in the Minitab programming code. The variable `lnclose` contains the values of `close` transformed by the natural logarithm. The variable `close9` has the same values as the variable `close`, except for missing values for patients 3 and 10. The questions can for most parts be answered independently of each other.

a) (2 points)

Characterize the study type (e.g., experimental or another type) and the statistical design (e.g., one-sample or another design).

It is of interest to estimate the probability that a skin graft survives for at least 30 days and to compare the rejection times of grafts with poor and close compatibility. Due to the incomplete observations, the poorly and closely matched grafts are initially considered separately.

b) (3 points)

For the grafts with poor compatibility, a calculation based on the normal distribution is desired. Describe the distribution of the rejection times on both original (untransformed) and natural-log transformed scale, and assess on which of the two scales (if any) the distribution might conform reasonably well to a normal distribution.

c) (2 points)

Still for the poorly matched grafts, use the normal distribution to estimate the probability of a skin graft surviving for at least 30 days; carry out the calculation on the scale where you found the data to be best approximated by a normal distribution.

d) (3 points)

For the grafts with close compatibility, estimate the probability of a skin graft surviving for at least 30 days, with an associated 95% confidence interval. Use as much of the data as possible, while taking into account the incomplete observations for patients 3 and 10.

e) (5 points)

Carry out a statistical analysis (parametric or nonparametric) of the data to investigate whether grafts with close compatibility generally survive longer than grafts with poor compatibility. Also here, use as much of the data as possible, while taking into account the incomplete observations for patients 3 and 10. Discuss (briefly) whether your way of dealing with the incomplete observations might have affected your results, and if so, how.

Minitab commands for Question 1 (from the History window) and output:

```
Name C5 'lnclose'  
Let 'lnclose' = ln('close')  
Name C6 'lnclose9'  
Let 'lnclose9' = ln('close9')  
Name C7 'lnpoor'  
Let 'lnpoor' = ln('poor')  
Name C8 'diff'  
Let 'diff' = 'close'-'poor'  
Name C9 'diff9'  
Let 'diff9' = 'close9'-'poor'  
Name C10 'diffln'  
Let 'diffln' = 'lnclose'-'lnpoor'  
Name C11 'diffln9'  
Let 'diffln9' = 'lnclose9'-'lnpoor'  
Print 'close'-'diffln9'.
```

Data

Row	close	close9	poor	Inclose	Inclose9	Inpoor	diff	diff9	diffln	diffln9
1	37	37	29	3.61092	3.61092	3.36730	8	8	0.24362	0.24362
2	19	19	13	2.94444	2.94444	2.56495	6	6	0.37949	0.37949
3	57	*	15	4.04305	*	2.70805	42	*	1.33500	*
4	93	93	26	4.53260	4.53260	3.25810	67	67	1.27450	1.27450
5	16	16	11	2.77259	2.77259	2.39790	5	5	0.37469	0.37469
6	23	23	18	3.13549	3.13549	2.89037	5	5	0.24512	0.24512
7	20	20	26	2.99573	2.99573	3.25810	-6	-6	-0.26236	-0.26236
8	43	43	43	3.76120	3.76120	3.76120	0	0	0.00000	0.00000
9	29	29	18	3.36730	3.36730	2.89037	11	11	0.47692	0.47692
10	60	*	42	4.09434	*	3.73767	18	*	0.35667	*
11	18	18	19	2.89037	2.89037	2.94444	-1	-1	-0.05407	-0.05407

Describe 'close' - 'diffln9';

Mean;
 StDeviation;
 QOne;
 Median;
 QThree;
 Minimum;
 Maximum;
 Skewness;
 N.

Statistics

Variable	N	Mean	StDev	Minimum	Q1	Median	Q3	Maximum	Skewness
close	11	37.73	24.02	16.00	19.00	29.00	57.00	93.00	1.35
close9	9	33.11	24.27	16.00	18.50	23.00	40.00	93.00	2.26
poor	11	23.64	10.88	11.00	15.00	19.00	29.00	43.00	0.88
Inclose	11	3.468	0.583	2.773	2.944	3.367	4.043	4.533	0.53
Inclose9	9	3.335	0.561	2.773	2.917	3.135	3.686	4.533	1.32
Inpoor	11	3.071	0.447	2.398	2.708	2.944	3.367	3.761	0.23
diff	11	14.09	21.68	-6.00	0.00	6.00	18.00	67.00	1.84
diff9	9	10.56	21.78	-6.00	-0.50	5.00	9.50	67.00	2.68
diffln	11	0.397	0.500	-0.262	0.000	0.357	0.477	1.335	1.03
diffln9	9	0.298	0.437	-0.262	-0.027	0.245	0.428	1.275	1.33