Extra exercise 4

Transmission of blood types in the ABO system. The blood type of the child is determined by the allele combination received from its parents. Within the pair of alleles of each parent one allele is randomly selected with probability 0.5.

(a,b) The resulting blood types for the possible allele combinations in parts (a) and (b) are shown the left and right tables below, respectively.

Part (a)		
First parent	Second parent A	Second parent B
A	A	AB
B	AB	B

Part (b)		
First parent	Second parent A	Second parent O
A	A	A
B	AB	B

Each allele combination has probability $0.5 \cdot 0.5 = 0.25$, by the multiplication rule (because of the independence of the alleles from the two parents). Therefore the blood type probability distributions have the probabilities:

(a) P(A) = 0.25, P(AB) = 0.5, P(B) = 0.25,

(b) P(A) = 0.5, P(AB) = 0.25, P(B) = 0.25.

For (b), the questions were not about the blood type distribution for the children, but about the blood types of two children. Let us denote their bloodtypes by random variables $X1$ and $X2$. Both $X1$ and $X2$ have the probability distribution listed above for (b).

The probability that both children have blood type A is computed by the multiplication rule (because their blood types are independent),

$$\mathrm{P}(X1 = \mathrm{A}, X2 = \mathrm{A}) = \mathrm{P}(X1 = \mathrm{A}) \cdot \mathrm{P}(X2 = \mathrm{A}) = 0.5 \cdot 0.5 = 0.25.$$

The probability that the two children have the same blood type is computed by adding up probabilities for all combinations of blood types that qualify,

$$\begin{aligned} \mathrm{P}(X1 = X2) &= \mathrm{P}(X1{=}X2{=}\mathrm{A} \text{ or } X1{=}X2{=}\mathrm{AB} \text{ or } X1{=}X2{=}\mathrm{B}) \\ &= \mathrm{P}(X1 = \mathrm{A}, X2 = \mathrm{A}) + \mathrm{P}(X1 = \mathrm{AB}, X2 = \mathrm{AB}) + \mathrm{P}(X1 = \mathrm{B}, X2 = \mathrm{B}) \\ &= 0.5 \cdot 0.5 + 0.25 \cdot 0.25 + 0.25 \cdot 0.25 = 0.375. \end{aligned}$$

(c) In (a), the blood type distribution actually corresponds to a binomial distribution with $n = 2$ and $p = 0.5$, because the blood type is effectively determined by counting the number of A alleles (values 2, 1, and 0, respectively, as listed above). If this is not obvious for you, try computing them from the binomial distribution (use Table 1 of Stevens, Table C of IPS, or Minitab).

In (b), the blood type distribution does not correspond to a binomial distribution, but the calculation of the probability of both children having blood type A does, again with $n = 2$ and $p = 0.5$. In contrast, the second probability is *not* from a binomial distribution.