

### Supplementary exercise 4.71 of IPS7e

A discrete probability distribution for the temperature ( $X$ ) of the flame in a glass heating process. (As an aside, one might think that a continuous distribution would be more natural.) The probability function is given by the values in the table below. We note that the probability distribution is valid because the probabilities are  $\geq 0$  and sum to 1 across the possible outcomes.

|  |      |      |      |      |      |
|--|------|------|------|------|------|
| Temperature $x$ ( $^{\circ}\text{C}$ ) | 540  | 545  | 550  | 555  | 560  |
| Probability $p(x)$                     | 0.10 | 0.25 | 0.30 | 0.25 | 0.10 |

- (a) We calculate the mean temperature as (letting  $X$  denote a random variable from this distribution):

$$E X = \sum_x x \cdot p(x) = 540 \cdot 0.1 + 545 \cdot 0.25 + 550 \cdot 0.3 + 555 \cdot 0.25 + 560 \cdot 0.1 = 550.$$

The distribution is (visibly) symmetrical around 550, so we could have guessed that the mean would equal 550.

We calculate the standard deviation for  $X$  in two steps: first the variance, and then the standard deviation:

$$\begin{aligned} \text{Var } X &= \sum_x (x - EX)^2 p(x) \\ &= (540 - 550)^2 \cdot 0.1 + (545 - 550)^2 \cdot 0.25 + (550 - 550)^2 \cdot 0.3 \\ &\quad + (555 - 550)^2 \cdot 0.25 + (560 - 550)^2 \cdot 0.1 \\ &= 10 + 6.25 + 0 + 6.25 + 10 = 32.5, \\ \text{sd } X &= \sqrt{\text{Var } X} = \sqrt{32.5} = 5.7. \end{aligned}$$

- (b) The new variable of interest is  $T = X - 550$ , i.e., a translation of the original variable  $X$  by the value  $(-550)$ . We can either use the rules for a translation, or the general rules for linear transformation of a random variable.

$$\begin{aligned} ET &= EX - 550 = 0 \quad (\text{i.e., we adjust the center by the translation}), \\ \text{sd } T &= \text{sd } X = 5.7 \quad (\text{i.e., no impact on spread by a translation}). \end{aligned}$$

When using the formulae on slide 3L-16, you should set  $b=1$  and  $a=(-550)$ .

- (c) Here the variable of interest is  $Y = 1.8 \cdot X + 32$ . We should use the mean and standard deviation rules for linear transformation of random variables (slide 3L-16), with  $a=32$  and  $b=1.8$ .

$$\begin{aligned} EY &= 32 + 1.8 \cdot EX = 32 + 1.8 \cdot 550 = 1022, \\ \text{sd } Y &= 1.8 \cdot \text{sd } X = 1.8 \cdot 5.7 = 10.26 \approx 10.3. \end{aligned}$$

We finally, on the next page, show Minitab commands to carry out these calculations after the distributions has been entered in suitable columns, labeled  $\mathbf{x}$  and  $\mathbf{p}(\mathbf{x})$ .

Minitab commands and output:

```

Name c1 "x"
Set 'x'
  1( 540 : 560 / 5 )1
End.
name c2 "p(x)"
Set 'p(x)'
  1( 0.1 0.25 0.3 0.25 0.1 )1
End.
Name C3 'meanx'
Let 'meanx' = 'x' * 'p(x)'
Sum 'meanx'.
Name C4 'varx'
Let 'varx' = ('x'-550)^2 * 'p(x)'
Sum 'varx'.
Name C5 'xF'
Let 'xF' = 1.8 * 'x' + 32
Name C6 'meanxF'
Let 'meanxF' = 'xF' * 'p(x)'
Sum 'meanxF'.
Name C7 'varxF'
Let 'varxF' = ('xF' - 1022)^2 * 'p(x)'
Sum 'varxF'.
Print 'x' 'p(x)' 'meanx' 'varx' 'xF' 'meanxF' 'varxF'.

```

|  |
|--|
| <b>Sum of meanx</b><br>Sum of meanx = 550    |
| <b>Sum of varx</b><br>Sum of varx = 32.5     |
| <b>Sum of meanxF</b><br>Sum of meanxF = 1022 |
| <b>Sum of varxF</b><br>Sum of varxF = 105.3  |

| Data Display |     |      |        |       |      |        |       |
|--------------|-----|------|--------|-------|------|--------|-------|
| Data         |     |      |        |       |      |        |       |
| Row          | x   | p(x) | meanx  | varx  | xF   | meanxF | varxF |
| 1            | 540 | 0.10 | 54.00  | 10.00 | 1004 | 100.40 | 32.40 |
| 2            | 545 | 0.25 | 136.25 | 6.25  | 1013 | 253.25 | 20.25 |
| 3            | 550 | 0.30 | 165.00 | 0.00  | 1022 | 306.60 | 0.00  |
| 4            | 555 | 0.25 | 138.75 | 6.25  | 1031 | 257.75 | 20.25 |
| 5            | 560 | 0.10 | 56.00  | 10.00 | 1040 | 104.00 | 32.40 |

Note that the Minitab calculator cannot be used for a simple (scalar) calculation such as:  $\sqrt{105.3} = 10.25$ . You would either have to use the calculator built into Windows or another calculator.