Supplementary exercise 9.52 of IPS7e

Data: A clinical trial (or randomized comparative experiment) on the therapeutic effect of gastric freezing on ulcers in the upper intestine. It may seem a bit surprising that such a trial was allowed to go ahead, with the ethical implications of randomizing patients to treatments; it must however have been conducted quite long ago. Out of $n_1 = 82$ patients that were subjected to gastric freezing, $X = 28$ improved. In the control group, $Y = 30$ out of $n_2 = 78$ patients improved.

Model: Two independent binomial distributions; in our notation, $X \sim \mathrm{B}(n_1, p_1)$ and $Y \sim \mathrm{B}(n_2, p_2)$.

Estimation: The sample proportions are: $\hat{p}_1 = X/n_1 = 28/82 = 0.341$ and $\hat{p}_2 = Y/n_2 = 30/78 = 0.385$.

(a) The hypothesis of interest is equal proportions in the two groups, with a two-sided alternative:

$$\text{null hypothesis } \mathrm{H}_0 : p_1 = p_2 \quad \text{versus} \quad \text{alternative } \mathrm{H}_a : p_1 \neq p_2.$$

One of our test statistics for two proportions is the classical z-test. Even though we could do the calculations manually from the information above, we let Minitab do the work; note that the Test method must be set to "Use the pooled estimate of the proportion".

```
MTB > PTwo 82 28 78 30;
SUBC>   Confidence 95.0;
SUBC>   Test 0.0;
SUBC>   Alternative 0;
SUBC>   Pooled.
```

Test and CI for Two Proportions

Method

p_1: proportion where Sample 1 = Event
p_2: proportion where Sample 2 = Event
Difference: $p_1 - p_2$

Descriptive Statistics

Sample	N	Event	Sample p
Sample 1	82	28	0.341463
Sample 2	78	30	0.384615

Estimation for Difference

Difference	95% CI for Difference
-0.0431520	(-0.192118, 0.105814)

CI based on normal approximation

Test

Null hypothesis	H_0: $p_1 - p_2 = 0$
Alternative hypothesis	H_1: $p_1 - p_2 \neq 0$

Method	Z-Value	P-Value
Normal approximation	-0.57	0.570
Fisher's exact		0.623

The pooled estimate of the proportion (0.3625) is used for the tests.

Comments:
The test statistic is $z = -0.57$ with a corresponding $P = 0.57$. We will address below whether the conditions for use of the z-test are met. Assuming this to be the case, we conclude that there is no evidence against H_0; we expand on the wording of the conclusion in (c) below.

(b) The model, estimation and hypotheses are the same; we are still comparing two independent proportions (or binomial distributions). The data have been provided in the preferred Minitab format for two-way tables. The Minitab listing below shows that the Pearson chi-square statistic equals $X^2 = 0.322 = (-0.57)^2 = z^2$, and indeed the P-value is exactly the same as above.

```
MTB > XTabs 'improv' 'tx';
SUBC>   Layout 1 1;
SUBC>   Frequencies 'count';
SUBC>   Counts;
SUBC>   ColPercents;
SUBC>   ChiSquare;
SUBC>   Expected;
SUBC>   DMissing 'improv' 'tx'.
```

Tabulated Statistics: improv, tx

Using frequencies in count

Rows: improv Columns: tx

	0	1	All
0	48	54	102
	61.54	65.85	63.75
	49.73	52.27	
1	30	28	58
	38.46	34.15	36.25
	28.27	29.73	
All	78	82	160
	100.00	100.00	100.00

Cell Contents
Count
% of Column
Expected count

Chi-Square Test

	Chi-Square	DF	P-Value
Pearson	0.322	1	0.570
Likelihood Ratio	0.322	1	0.570

Comments:
The listing of the two-way table with the expected counts (under H_0) show that the conditions for use of the z- and X^2-tests are met: all expected cell counts are (clearly) above 5.

(c) Conclusion: We cannot reject H_0. Considering the large P-value and the almost equal sample proportions, there is no substantial indication and certainly no evidence whatsoever that the treatment does better than control. Therefore the study can be said to suggest that gastric freezing is a pretty useless treatment.