Supplementary exercises 7.127 and 7.129 of IPS7e

Data: Birth weight (in g) of babies born by women who tested positive on cocaine use compared to women who did not test positive. Note that this must be an observational study, not an experiment. Assume $n = 75$ women in each group.

Model: Two-sample (independent) inference based on normal distributions $N(\mu_1, \sigma)$ and $N(\mu_2, \sigma)$; note that we assume the standard deviation to be the same in the two groups (a convenient assumption for planning purposes, but if in reality they differed, it would be good to take more subjects in the group with the highest standard deviation). The standard deviation is considered unknown and to be estimated by the data, and our guess for the (common) value is $\sigma = 650$.

The hypotheses of interest are:

* $H_0 : \mu_1 = \mu_2$ (no difference between groups),
* $H_a : \mu_1 \neq \mu_2$ (different birth weights in the cocaine and control groups; a one-sided alternative can be argued as well but there might be so much doubt about the results from the previous study that the researcher opts for a more cautious approach).

Exercise 7.127
We use Minitab for a power calculation assuming a true population mean difference of $\mu_1 - \mu_2 = 350$.

```
MTB > Power;
SUBC>   TTwo;
SUBC>     Sample 75;
SUBC>     Difference 350;
SUBC>     Sigma 650.
```

Power and Sample Size

2-Sample t Test
Testing mean 1 = mean 2 (versus ≠)
Calculating power for mean 1 = mean 2 + difference
α = 0.05 Assumed standard deviation = 650

Results

Difference	Sample Size	Power
350	75	0.905905

The sample size is for each group.

Comments
The computed power is 0.91 against a two-sided alternative (and 0.95 against a one-sided alternative; not shown). A study of this size is pretty likely to find a significant result if the true magnitude of the difference is as presumed.

Exercise 7.129
Our solution is for $n = 75$ only. The 95% confidence interval for $\mu_1 - \mu_2$ is given by (when using the pooled standard deviation s_p):

$$\bar{X}_1 - \bar{X}_2 \pm t^* \cdot s_p \sqrt{1/n_1 + 1/n_2},$$

where $n_1 = n_2 = 75$, $\text{df} = (n_1 - 1) + (n_2 - 1) = 148$ and therefore $t^* = t_{0.975}(148) \approx t_{0.975}(100) = 1.984$ from our t-distribution table (Minitab gives the exact value as 1.976), so finally the margin of error is

$$1.984 \cdot 650\sqrt{2/75} = 210.6 \approx 210.$$

It is seen that an observed difference of 350 would make the CI very far from including zero, and would therefore correspond to a strongly significant result. This may seem surprising in view of the power in the range 0.9-0.95 obtained with $n = 75$. However, a power calculation involves the true population difference, not the observed sample difference. The true population difference being 350 does not all guarantee that the sample difference is also close to 350, therefore it is a *stronger* requirement to have a fairly high power at a certain population difference than having a margin of error for the CI of the same size.

Another way of saying this is that controlling the margin of error leads to a statement about what an observed mean difference of a certain size, here 210 g, between the two groups tells us (namely, it is just at the cut-off for significance at $P < 0.05$), whereas a power calculation gives us the probability of getting a significant result from the two samples when the true population difference is 350 g.

We conclude with some Minitab commands and output, first to calculate t^* in $t(148)$, and next a way to display the margin of error for the confidence interval in the scenario studied (by offering fictitious mean values for the two groups and reading of the margin of error from the CI).

```
MTB > InvCDF .025;
SUBC>   T 148.
```

Inverse Cumulative Distribution Function

Student's t distribution with 148 DF

P(X ≤ x)	x
0.025	-1.97612

```
MTB > TwoT 75 0 650 75 0 650;
SUBC>   Confidence 95.0;
SUBC>   Test 0.0;
SUBC>   Alternative 0;
SUBC>   Pooled.
```

Two-Sample T-Test and CI

Method

μ_1: mean of Sample 1
μ_2: mean of Sample 2
Difference: $\mu_1 - \mu_2$

Equal variances are assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Sample 1	75	0	650	75
Sample 2	75	0	650	75

Estimation for Difference

Difference	Pooled StDev	95% CI for Difference
0	650	(-210, 210)

Test

Null hypothesis $H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

T-Value	DF	P-Value
0.00	148	1.000